

Broken Symmetries in the Early Universe*

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Abstract

The year is 10^{10} B.C. All the symmetries of Nature broken at low temperatures are completely restored. All of them?

No!

A tiny space of parameters, near the nonperturbative region, is there to resist now and ever to the invading forces of symmetry restoration. And life is not easy for the thermally produced strings, monopoles and domain walls...

I. INTRODUCTION

Both common sense and daily life experience suggest the existence of phase transitions in systems exposed to temperature changes, leaving one with the belief that a hotter environment normally implies more symmetry. And yet there are counterexamples, such as the Rochelle salt, which actually exhibit contrary behavior. It has been known now for some time that in particle physics systems, at least in theories beyond the standard model, the question of symmetry patterns at high temperature is rather complex, and depends on the parameter space of the theory under discussion. It is our aim here to provide a short (and still somewhat pedagogical) review of this phenomenon.

Our motivation is at least twofold. The main reason for this study is simply curiosity; one wants to know if the complicated systems of present-day particle physics mimic more familiar ones, such as water or a ferromagnet. However, our interest is not purely academic. The physics of the standard model of electro-weak interactions is based on the idea of spontaneous symmetry breaking, and thus what happens at high T could in principle help to probe the nature of this mechanism. Now, the relevant temperatures are too high to be of direct laboratory significance, but on the other hand the early universe can serve as an ideal place

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to study this important issue. The cosmological implications of high temperature symmetry behavior are profound and have to deal with such central questions as baryogenesis, the monopole and domain wall problems, the dynamics of cosmic strings, etc.

Before addressing this issue in detail, we wish to say a few words about the early work in the field. The original work by Kirzhnits and Kirzhnits and Linde [1], suggested that at sufficiently high temperatures spontaneously broken symmetries are restored in a phase transition. This conclusion has been strengthened in the classic papers of Weinberg [2] and Dolan and Jackiw [3], but remarkably enough, already then Weinberg notes the possibility of symmetry nonrestoration at high T in theories with more than one Higgs multiplet (he cites Coleman as being behind this observation). Interestingly, this went unnoticed for some years until the work of Mohapatra and Senjanović [4]. They were mainly motivated by the question of spontaneous CP violation at high temperatures for the sake of baryogenesis, and found out that it was possible to keep CP broken in a multi-Higgs $SU(2) \times U(1)$ model (recall that the idea of spontaneous CP violation requires necessarily more than one $SU(2) \times U(1)$ Higgs doublet). Much to their surprise, symmetry nonrestoration, as we have said, had already been discussed by Weinberg.

In [4] it was also pointed out that symmetry nonrestoration may provide a way out of the domain wall problem, without fully addressing the question though. Soon after them Langacker and Pi [5] pointed out that the same phenomenon may provide a way out of the monopole problem if electromagnetic gauge $U(1)$ invariance were to be broken in the early universe. However, the examples provided by all the above were in some sense *ad hoc*, since they were achieved by enlarging the the minimal models just to serve this purpose.

Recently, the issue of high T symmetry behavior was readdressed in a series of papers [6–8] devoted to the minimal, already accepted particle physics models with special emphasis on the domain wall and monopole problems. Here we review the central results of this study. In the next section, after a general discussion on symmetry nonrestoration, we give some examples on how it can be realized in the context of global and local symmetries, paying particular attention to some interesting examples: spontaneous CP violation, Peccei-Quinn symmetry, and $SU(5)$ GUT. Then in section III, we show how this is related to topological defects production and how it can lead to a solution of the monopole and domain wall problems in those theories. The final section contains a brief outlook.

II. BROKEN SYMMETRIES AT HIGH TEMPERATURE

The issue of what happens to a spontaneously broken symmetry when temperature effects are taken into account was addressed many years ago [1–3]. When the temperature reaches values much bigger than the Higgs field mass, its effects can be accounted for (up to the one-loop level) by a mass term in the effective potential proportional to T^2 . More precisely, for a general Higgs potential written in terms of N real fields φ_i , the temperature contribution for $T \gg m_\varphi$ is

$$\Delta V(T) = \frac{T^2}{24} \left[\left(\frac{\partial^2 V}{\partial \varphi_i \partial \varphi^i} \right) + 3(T_a T_a)_{ij} \varphi^i \varphi^j \right] \quad (1)$$

where sum over repeated indices is assumed. This term being positive, it would unavoidably imply that a critical temperature will be reached above which the mass term for the Higgs is positive, restoring the symmetry. While this is certainly true for theories involving only one Higgs, when two or more fields are responsible for the symmetry breaking, this need not be the case [2,4]. Consider for example a simple such theory, with a $U(1) \times U(1)$ global symmetry and two complex Higgs fields ϕ and χ and a potential

$$V = -\frac{m_\phi^2}{2} \phi^* \phi + \frac{\lambda_\phi}{4} (\phi^* \phi)^2 - \frac{m_\chi^2}{2} \chi^* \chi + \frac{\lambda_\chi}{4} (\chi^* \chi)^2 + \frac{\alpha}{2} \phi^* \phi \chi^* \chi \quad (2)$$

and calculate the effective masses at high temperature using (1)

$$\begin{aligned} m_\phi^2(T) &= -m_\phi^2 + \frac{T^2}{12} (2\lambda_\phi + \alpha) \equiv -m_\phi^2 + T^2 \nu_\phi^2 \\ m_\chi^2(T) &= -m_\chi^2 + \frac{T^2}{12} (2\lambda_\chi + \alpha) \equiv -m_\chi^2 + T^2 \nu_\chi^2 \end{aligned} \quad (3)$$

The crucial point is that the coupling constant α enters the mass terms at high temperature. Nothing forces α to be positive, all that is required is that the potential (2) is bounded from below, which implies

$$\lambda_\phi \lambda_\chi > \alpha^2 \quad (4)$$

One can have $\alpha < 0$, and for example $\lambda_\phi > 2|\alpha| > 4\lambda_\chi$. Then ν_χ in (3) is negative, and $m_\chi(T)$ is negative for all temperatures. Notice that (4) prevents us from taking *both* ν_χ and ν_ϕ negative. Then one of the the $U(1)$ groups is broken for any value of T. One can conceive of a model in which there is only one $U(1)$ symmetry, by including the term

$$\beta_1 \phi^* \chi \chi^* \chi + \beta_2 \chi^* \phi \phi^* \phi + h.c. \quad (5)$$

in the potential. In this case, the cubic terms will force both of the vev's to be nonzero, even if only one of the masses is negative. The $U(1)$ symmetry is completely broken at high temperature, for the same range of parameters as before.

The question of restoration becomes then a *dynamical* one, depending on the parameters of the potential.

A. Global symmetries: $O(N_1) \times O(N_2)$ model

It is not difficult to convince oneself that nonrestoration of symmetries is also possible in more complicated and realistic theories. Suppose that the fields in the previous example transform under more complicated groups. One would have then a bigger variety of possible self-couplings and couplings with the other field, introducing a number of coupling constants. The conditions of boundedness of the potential analogous to (4) can be many, and very complicated. However, it is enough that nonrestoration occurs for a reasonable range of parameters, so it is perfectly natural to ask for some of the couplings to be small. Then one can consider only those couplings analogous to the ones of the simple model. That is, for

fields (Φ, Ξ) transforming under the representations R_1, R_2 of some group G containing N_1, N_2 real fields (ϕ, χ) , write the Higgs potential as

$$V = \sum_{a=1}^{N_1} \sum_{b=1}^{N_2} \left\{ -\frac{m_\phi^2}{2} \phi^a \phi_a + \frac{\lambda_\phi}{4} (\phi^a \phi_a)^2 - \frac{m_\chi^2}{2} \chi^b \chi_b + \frac{\lambda_\chi}{4} (\chi^b \chi_b)^2 - \frac{\alpha}{2} \phi^a \phi_a \chi^b \chi_b \right\} + V_s \quad (6)$$

where V_s contains terms whose coupling constants are assumed to be much smaller than $\lambda_\phi, \lambda_\chi$ and α . Thus in this case the symmetry is $O(N_1) \times O(N_2)$. We will use the $O(N_1) \times O(N_2)$ models as a prototype that can effectively mimic more complicated groups.

Taking $\alpha < 0$, the condition for the boundedness of the potential is again (4). The high temperature contributions to the masses are

$$\begin{aligned} \Delta m_\phi^2(T) &= T^2 \nu_\phi^2 = T^2 \left[\lambda_\phi \left(\frac{2+N_1}{12} \right) - \frac{N_2}{12} \alpha \right] \\ \Delta m_\chi^2(T) &= T^2 \nu_\chi^2 = T^2 \left[\lambda_\chi \left(\frac{2+N_2}{12} \right) - \frac{N_1}{12} \alpha \right] \end{aligned} \quad (7)$$

and the G symmetry will not be restored if the couplings lie in the range

$$\lambda_\phi > \left(\frac{2+N_2}{N_1} \right) \alpha > \left(\frac{2+N_2}{N_1} \right)^2 \lambda_\chi \quad (8)$$

Some relevant features of the range (8) are worth mentioning.

- notice that there is no lower bound on the smallest coupling, so one can always take it small enough to avoid the danger of the couplings getting too large and in conflict with perturbation theory. This is not the case if G is a gauge symmetry, since the gauge coupling will have to enter in the discussion, as we will see later.
- the conditions are weaker if the ratio N_2/N_1 is big, that is, it will be easier for the representation with fewer real fields to maintain its vev at high temperature.

In the simple example considered, only one of the fields can have a vev. This means that any subgroup of G preserved by its vev will be restored. But condition (4) only prevents us from taking both mass terms negative, and with an adequate coupling one can have both vevs nonzero even if one of the masses is positive. This will be the case if for example the symmetry allows for terms of the type $\phi^3 \chi$, as we saw in the general example. It is possible then to keep G completely broken. We will illustrate how the mechanism can actually work by considering two examples: a discrete symmetry (CP) and a global U(1) symmetry (Peccei-Quinn).

1. Spontaneous CP violation

Generally speaking, models of spontaneous symmetry breaking cannot be analyzed as suggested before, by taking some of the couplings to be negligible and considering only two self-couplings and a mixed one. For instance, in T.D. Lee's [9] original model of spontaneous CP violation, the CP-violating phase is the relative phase between the vevs of two doublet fields Φ_1, Φ_2 , and it appears due to the presence of terms of the type $\Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_2$. To have CP nonrestored, it is not enough to keep the vev's nonzero, one has also to make sure that

the phase persists. In [8] it has been shown that neither the T. D. Lee model, nor the model of CP violation with three doublets [10], allow for nonrestoration of CP.

There is however one model of spontaneous CP violation which in addition has the nice feature of providing natural flavor conservation, where nonrestoration is easily achieved [11,8]. It is a minimal extension of the Standard Model with the addition of a singlet field (S), odd under CP, and an additional down quark, with both left and right components D_L^a and D_R^a , singlets under $SU(2)$.

The interaction Lagrangian for the down quarks, symmetric under CP, contains the terms

$$\begin{aligned}\mathcal{L}_Y = & (\bar{u}\bar{d})_L^a h_a \Phi D_R + (\bar{u}\bar{d})_L^a h_{ab} \Phi d_R^b \\ & + M_D \bar{D}_L D_R + M_a (\bar{D}_L d_R^a + h.c.) \\ & + i f_D S (\bar{D}_L D_R - \bar{D}_R D_L) + i f_a S (\bar{D}_L d_R^a - \bar{d}_R^a D_L)\end{aligned}\quad (9)$$

Clearly, when S gets a vev (at a scale σ above the weak scale M_W) CP is spontaneously broken by the terms in the last line. CP violation at low energies is then achieved by complex phases appearing in the CKM matrix through the mixings of d and D quarks.

The most general potential for the fields Φ and S can be written as

$$\begin{aligned}V(\Phi, S) = & -m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 \\ & -\frac{m_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 - \frac{\alpha}{2} \Phi^\dagger \Phi S^2\end{aligned}\quad (10)$$

and it has a minimum at

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad ; \quad \langle S \rangle = \sigma \quad (11)$$

We can use here the general equations of the previous section, (4), (7) and (8) with $N_1 = 4$ and $N_2 = 1$. Notice that although the high-T mass of the doublets will contain the gauge coupling, it will not appear in the conditions upon the coupling constants, since we will only require that the mass of the singlet is negative at high T. This can be achieved if the couplings fall in the range

$$\lambda_\Phi > \frac{3}{2} \alpha > \left(\frac{3}{2}\right)^2 \lambda_S \quad (12)$$

Thus CP can be violated at all temperatures.

2. Peccei-Quinn symmetry

A very illustrative example of nonrestoration of a physically relevant symmetry is that of the $U(1)_{PQ}$ global symmetry, whose spontaneous breakdown provides a solution to the strong CP problem [12]. In the invisible axion version [13] of the Peccei-Quinn mechanism, $U(1)_{PQ}$ is broken down at a scale M_{PQ} much bigger than the QCD scale by the vev of a singlet field. The model requires in addition two doublet fields (ϕ_1, ϕ_2) that will couple to the quarks. Under $U(1)_{PQ}$ they transform as

$$\phi_1 \rightarrow e^{i\alpha} \phi_1 \ ; \ \phi_2 \rightarrow e^{-i\alpha} \phi_2 \ ; \ S \rightarrow e^{2i\alpha} S \quad (13)$$

The Higgs potential, invariant under $SU(2)_L \times U(1)_Y \times U(1)_{PQ}$ is written

$$\begin{aligned} V_{PQ} = \sum_i \left[-\frac{m_i^2}{2} \phi_i^\dagger \phi_i + \frac{\lambda_i}{4} (\phi_i^\dagger \phi_i)^2 \right] - \frac{\alpha}{2} (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) - \frac{\beta}{2} (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\ - \frac{m_s^2}{2} S^* S + \frac{\lambda_s}{4} (S^* S)^2 - \sum_i \left(\frac{\gamma_i}{2} \phi_i^\dagger \phi_i \right) S^* S - M (\phi_1^\dagger \phi_2 S + \phi_2^\dagger \phi_1 S^*) \end{aligned} \quad (14)$$

For $\beta > 0$, the minimum is found at

$$\langle \Phi_i \rangle = \begin{pmatrix} 0 \\ v_i \end{pmatrix} \ ; \ \langle S \rangle = v_S \quad (15)$$

To have $U(1)_{PQ}$ broken at any temperature, it is enough to keep the vev of the singlet nonzero for all T, and as before, we can use the formulas for the global case. The high temperature mass for S is

$$m_S^2(T) = -m_S^2 + \frac{T^2}{3} (\lambda_S - \gamma_1 - \gamma_2) \quad (16)$$

But since we have three fields in this model, conditions (4) have to be generalized. Taking $v_S \gg v_i$, they are, to leading order

$$\lambda_i > 0 \ , \ \lambda_S > 0 \ ; \ \lambda_i \lambda_S > \gamma_i^2 \ ; \ \lambda_1 \lambda_2 > (\alpha + \beta)^2 \quad (17)$$

$$\begin{aligned} M v_s^3 \left[\frac{v_1^3}{v_2} (\lambda_1 \lambda_S - \gamma_1^2) + \frac{v_2^3}{v_1} (\lambda_2 \lambda_S - \gamma_2^2) - 2 v_1 v_2 (\lambda_S (\alpha + \beta) + \gamma_1 \gamma_2) \right] \\ + v_S^2 v_1^2 v_2^2 \left[\lambda_1 \lambda_2 \lambda_S - \lambda_1 \gamma_2^2 - \lambda_2 \gamma_1^2 - \lambda_S (\alpha + \beta)^2 - 2 \gamma_1 \gamma_2 (\alpha + \beta) \right] > 0 \end{aligned} \quad (18)$$

It is easily proven that it is possible to require $\gamma_1 + \gamma_2 > \lambda_S$, thus having a negative mass for S , without contradicting conditions (17) and (18). One can then have $U(1)_{PQ}$ broken at arbitrarily high temperatures.

B. Gauged case

As we have already mentioned, when the symmetry is gauged nonrestoration is not straightforward. The gauge coupling provides a lower bound on the coupling constants, and depending on the particular gauge group chosen, one can then have to require the coupling constants to be of order one, away from the perturbative regime.

To see it explicitly, consider a simplified model as the one of section II A, that is one where only the relevant coupling constants are taken into account, and now the group G is gauged. The two fields Φ and Ξ transform under the representations R_i ($i = 1, 2$) whose generators satisfy

$$Tr(T_i^a T_i^b) = c_i \delta^{ab} \quad (19)$$

Then the high-temperature masses are

$$\begin{aligned}\Delta m_\phi^2(T) &= T^2 \nu_\phi^2 = T^2 \left[\lambda_\phi \left(\frac{2+N_1}{12} \right) - \frac{N_2}{12} \alpha + \frac{1}{4} g^2 \frac{Dim(G)}{N_1} r_1 c_1 \right] \\ \Delta m_\chi^2(T) &= T^2 \nu_\chi^2 = T^2 \left[\lambda_\chi \left(\frac{2+N_2}{12} \right) - \frac{N_1}{12} \alpha + \frac{1}{4} g^2 \frac{Dim(G)}{N_2} r_2 c_2 \right]\end{aligned}\quad (20)$$

where g is the gauge coupling, $Dim(G)$ is the dimension of the group and r_i is 1 when the representation contains real fields, 2 when it is complex. Asking ν_χ to be negative and at the same time the fulfillment of the bound (4) now implies

$$\lambda_\phi > \frac{\alpha^2}{\lambda_\chi} > \frac{1}{\lambda_\chi} \left[\left(\frac{N_2+2}{N_1} \right) \lambda_1 + 3g^2 \frac{Dim(G)}{N_1 N_2} r_2 c_2 \right]^2 \quad (21)$$

As a function of λ_χ , λ_ϕ has a minimum at

$$\lambda_\chi = 3g^2 \frac{Dim(G)}{N_2(2+N_2)} r_2 c_2 \quad (22)$$

So λ_ϕ is bounded from below as

$$\lambda_\phi > 12g^2 \frac{(2+N_2)Dim(G)r_2 c_2}{N_1^2 N_2} \quad (23)$$

The dimension of the representation R_1 (under which the fields that loses its vev transforms) now plays an even more fundamental role: it has to be big enough, if we want perturbation theory to be valid. This is better illustrated by a concrete example

1. $SU(5)$ and nonrestoration

Being the simplest of GUTs, it is only natural to investigate the high temperature behavior of $SU(5)$. The usual pattern of symmetry breaking goes through the Standard Model, as

$$SU(5) \xrightarrow{\langle H \rangle} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\langle \Phi \rangle} SU(3)_C \times U(1)_{em}$$

where H is taken to transform under the adjoint representation, while Φ can be either in the 5-dimensional fundamental representation or, if one requires a realistic theory of fermion masses, in the 45-dimensional representation.

In ref. [14], a range of parameters was considered for which Φ (in the 5-dimensional representation) keeps its vev at high T , thus preventing the restoration of $SU(2)_L \times U(1)_Y$ of the Standard Model. Here we consider the case in which H keeps its vev, a case that may have interesting cosmological consequences [7], and that will be particularly illustrative.

First suppose that Φ is in the five-dimensional representation. Then in (23), setting $N_1 = 10$, $N_2 = 24$, $c_2 = 5$, $r_2 = 1$, we get

$$\lambda_\phi > \frac{78}{5} g^2 \quad (24)$$

For a typical value of $g^2 \sim 1/4$, we find $\lambda_\phi/4$ dangerously close to one. On the other hand, if we take the more realistic model where Φ is in the 45-dimensional representation, we have $N_1 = 90$, and then the lower limit is 9^2 smaller

$$\lambda_\phi > \frac{26}{135}g^2 \quad (25)$$

So it will be safe to take $\lambda_\phi \sim 0.05$. It is then possible to have $SU(5)$ broken at high temperatures.

III. TOPOLOGICAL DEFECTS

Symmetry nonrestoration can be used in certain theories to cure the problems related to topological defects. Topological defects such as monopoles, strings and domain walls, can arise in cosmological phase transitions, which are a direct consequence of symmetry restoration at high temperature. Namely, if symmetries are restored by thermal effects, one has a picture in which they become broken as the universe cools down. The fact that the Higgs field is only causally correlated inside a finite region at a given time, then, gives rise to defects via the so-called Kibble mechanism [15]

Of the three kinds of defects mentioned, only cosmic strings are compatible with the standard cosmology. Monopoles are produced in a phase transition in too big numbers [16], and domain walls are too heavy [17]: in both cases the result is that they overclose the universe.

As was suggested in [18,6,7], one way out could be to avoid the phase transition. In theories with more than one Higgs fields, this can be done in principle by requiring that the parameters of the potential fall into the ranges where nonrestoration, if possible, can occur.

The theories exhibiting nonrestoration considered in the previous section, are of the kind that admit topological defects. The theory of CP violation, based on a spontaneously broken discrete symmetry, has domain wall solutions. The global $U(1)$ symmetry of Peccei-Quinn allows for the formation of global strings, however when the QCD scale is reached, the Nambu-Goldston boson associated with its breaking (the axion) acquires a vev. When this happens, the strings become the edges of domain walls, which are stable. Finally, when $SU(5)$ breaks down to $SU(3) \times SU(2) \times U(1)$, monopoles are produced. As we have seen, there is a natural way to avoid the restoration of the symmetries at high temperature, i.e., to avoid the phase transition. Defects are then not produced via the Kibble mechanism.

However, it is still true that the theory admits the classical solutions that we call defects. In order not to actually have these structures formed, we have to make sure, to start with, that “initially”, i.e. at the Planck scale, the field is distributed uniformly over scales that are not causally correlated, at least over a scale of the size of the comoving horizon. This is the same as requiring that the so-called horizon problem be solved, for example by invoking an era of primordial inflation. But even if this condition is satisfied, thermal fluctuations can drive the field away from the minimum chosen. So one has to take into account the possibility of thermal production of defects, as we do now.

A. Domain Wall problem

Consider the nucleation of a large spherically symmetric domain wall or a closed loop of string. The production rate per unit time per unit volume at a temperature T will be given by [19]

$$\Gamma = T^4 \left(\frac{S_3}{2\pi T} \right)^{3/2} e^{-S_3/T} \quad (26)$$

where S_3 is the energy of the closed defect. The suppression factor $e^{-S_3/T}$ is readily calculated in the limit where the defect's radius is much bigger than its width. For the domain walls produced in the model of CP violation with a singlet, we get

$$\frac{S_3}{T} \gg \frac{16\pi}{3\sqrt{6}} \frac{\sqrt{2\alpha - 3\lambda_S}}{\lambda_S} \quad (27)$$

Analogously, for the Peccei-Quinn model the thermal production of large loops of strings is suppressed by

$$\frac{S_3}{T} \gg 4\pi^2 \frac{\sqrt{\gamma_1 + \gamma_2 - \lambda_S}}{\lambda_S} \quad (28)$$

We see that in both cases, it suffices to take the singlet's self-coupling λ_S small to avoid significant thermal production of defects.

B. Monopole problem

Monopoles can be thermally produced in e^+e^- (and other charged particles) collisions. Turner [20] has investigated the conditions under which the density of thermally produced monopoles will be consistent with cosmology, and found that we should have

$$\frac{m_M}{T} \geq 35 \quad (29)$$

where m_M is the monopole mass. More precisely, for $m_M/T \geq 20$, he that

$$\frac{n_M}{n_\gamma} \simeq 3 \times 10^3 \left(\frac{m_H}{T} \right)^3 e^{-2m_M/T} \quad (30)$$

where n_γ is the photon density; and from the upper limit $n_M/n_\gamma \leq 10^{-24}$, one obtains (29)

Now, in $SU(5)$ the lightest monopoles weigh [21]

$$m_M = \frac{10\pi}{\sqrt{2}g} v_H \quad (31)$$

For $g^2/(4\pi) \simeq 1/50$ or $g \simeq 1/2$, $m_M \simeq 40v_H$, and thus the consistency with the cosmological bound (29) implies

$$\frac{v_H}{T} \geq 1 \quad (32)$$

Obviously this will put even more restrictive conditions on the parameters of the theory. For the simplified $O(N_1) \times O(N_2)$ models we have considered (with $N_2 = 24$ and $N_1 = 10$ or 90), we have at high temperature

$$\frac{v_H^2}{T^2} = -\frac{\nu_H^2}{\lambda_H} > 1 \quad (33)$$

instead of just $\nu_H^2 > 0$. Condition (23) becomes now

$$\lambda_\phi > 12g^2 \frac{(N_2 + 14) \text{Dim}(G) c_2}{N_1^2 N_2} \quad (34)$$

We have for the case in which Φ is in the 45-dimensional representation

$$\lambda_\phi > \frac{38}{135} g^2 \quad (35)$$

which is perfectly compatible with perturbation theory still. We conclude that thermally produced monopoles can be kept below the density limit required by cosmology.

IV. OUTLOOK

We have illustrated how spontaneously broken symmetries may and may not be restored at high temperature. We have also suggested that symmetry nonrestoration may provide a way out of the domain wall and monopole problems. The latter may even be cured in the canonical $SU(5)$ theory, especially if one accepts the necessity of a 45 of Higgs to reproduce correctly the quark and lepton masses.

Our discussion so far has been based only on the leading one-loop computation of high temperature scalar masses. The situation becomes more complicated when the next-to-leading effects are included, as recently pointed out by Bimonte and Lozano [22] and Roos [23]. Bimonte and Lozano even find out that the $SU(5)$ example discussed above may be in trouble; more precisely that one may be taken out of the perturbative regime. We feel that more study is needed before one has a conclusive answer on these issues, but we should add that if they are right, one would be forced to turn one's attention to more complicated (and possibly more realistic) theories such as $SO(10)$, characterized by multistage symmetry breaking patterns. The work on this is now in progress.

We have also left out the supersymmetric theories. Here unfortunately we have a no-go theorem due to Mangano and Haber [24] which states that internal symmetries in the context of SUSY are necessarily restored at high T . As we were preparing this for print, a paper of Dvali and Tamvakis [25] has appeared which tries to offer a possible way out using higher dimensional nonrenormalizable interaction.

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